Milford Public Schools Curriculum

Department: Mathematics
Course Name: Calculus
Course Description:

UNIT # 1

Unit Title: Limits, Continuity, and Definition of the Derivative
Unit Description:
The idea of limits is important for discovering important ideas and definitions in calculus. Students should be able to compute various limits, including one-sided limits, limits at infinity, and infinite limits. They should work with tables and graphs in order to estimate the limit of a function at a point, as well as the algebraic techniques in order to find the exact limit of a function at a point. Students will then make the connection between limits and continuity, and finally develop the definition of the derivative using the concept of limits and use the derivative to compute the instantaneous rate of change of a function.

LEARNING GOALS

Enduring Understanding(s):
- The concept of a limit can be used to understand the behavior of functions.
- Continuity is a key property of functions that is defined using limits.
- The derivative of a function is defined as the limit of a difference quotient.

Essential Question(s):
- How can we decide what technique to use in order to find the limit of a function?
  - What processes do we need to carry out in order to do so?
- What conditions must be satisfied in order for a function to be continuous?
- How can we determine the instantaneous rate of change of a function using limits?
- What conditions must be satisfied in order for a function to be differentiable?

Content and Skills:
Students will be able to:
- Find limits of functions graphically and numerically
- Use the properties of limits to evaluate the limits of functions
- Use different analytic techniques to evaluate the limits of functions
- Evaluate one-sided limits
- Recognize unbounded behavior of functions
- Determine the continuity of functions
- Determine the continuity of a function on a closed interval
- Identify tangent lines to a graph at a point.
- Approximate the slopes of tangent lines to graphs at points.
- Use the limit definition to find the slopes of graphs at points.
- Use the limit definition to find the derivatives of functions.
- Describe the relationship between differentiability and continuity.

Standards Addressed: (Learning Objectives from the AP Calculus AB Course and Exam Description)
LO 1.1.A(a): Express limits symbolically using correct notations.
LO 1.1A(b): Interpret limits expressed symbolically.
LO 1.1B: Estimate limits of functions.
LO 1.1C: Determine limits of functions.
LO 1.1D: Deduce and interpret behavior of functions using limits.
LO 1.2A: Analyze functions for intervals of continuity or points of discontinuity.
LO 2.1A: Identify the derivative of a function as the limit of a difference quotient.

UNIT # 2

Unit Title: Derivative Rules
Unit Description: Using derivatives to describe rates of change allows students to understand change in a variety of contexts. Prior to this unit, students have already built the derivative using concepts of limits. Students will now apply various derivative rules and properties to functions in order to find their derivatives. In addition, students should be able to find the slope of a tangent line to a graph at a point, find derivatives implicitly, and solve for higher order derivatives.

LEARNING GOALS

Enduring Understanding(s):
- A function's derivative, which itself is a function, can be used to understand the behavior of the function.
- The derivative has multiple interpretations and applications including those that involve the instantaneous rates of changes.

Essential Question(s):
- How can we decide what technique or rules to use in order to find the derivative of a function?
  - What processes do we need to carry out in order to do so?
- How can we simplify derivatives?
- How can we apply the chain rule to find derivatives of functions which are defined implicitly?

Content and Skills:
Students will be able to:
- Find the derivatives of functions using the Constant Rule.
- Find the derivatives of functions using the Power Rule.
- Find the derivatives of functions using the Constant Multiple Rule.
- Find the derivatives of functions using the Sum and Difference Rules.
- Find the instantaneous rates of change of functions at points.
- Find the derivatives of functions using the Product Rule.
- Find the derivatives of functions using the Quotient Rule.
- Find derivatives using the Chain Rule.
- Find derivatives using the General Power Rule.
- Write derivatives in simplified form.
- Find derivatives explicitly.
- Find derivatives implicitly.
- Find higher order derivatives.

Standards Addressed:
LO 2.1C: Calculate derivatives.
LO 2.1D: Determine higher order derivatives.
LO 2.3B: Solve problems involving slope of a tangent line.

UNIT # 3

Unit Title: Applications of Derivatives
Unit Description:

Students will now apply all of the derivative rules they mastered in the previous unit in order to mathematically describe and solve for the rate of change of one variable with respect to another variable in a variety of contexts. Students should become familiar with a variety of real-world applications, including related rates, optimization, and particle motion. Students will also use derivatives to examine the behavior of functions, including: intervals of increase and decrease, maximums and minimums, intervals of concavity, and inflection points.

LEARNING GOALS

Enduring Understanding(s):

- The derivative has multiple interpretations and applications including those that involve instantaneous rates of change.

Essential Question(s):

- How can derivatives be used to analyze the properties of a function?
- How can derivatives be used to solve problems involving related rates, optimization, and rectilinear motion (velocity, speed, and acceleration)?

Content and Skills:

Students will be able to:

- Use derivatives to answer questions about real life situations.
- Examine related variables.
- Solve related-rate problems.
- Test for increasing and decreasing functions.
- Find the critical numbers of functions and find the open intervals on which functions are increasing or decreasing.
- Use increasing and decreasing functions to model and solve real-life problems.
- Recognize the occurrence of relative extrema of functions.
- Use the First-Derivative Test to find the relative extrema of functions.
- Find absolute extrema of continuous functions on a closed interval.
- Find minimum and maximum values of real-life models and interpret results in context.
- Determine the intervals on which the graphs of functions are concave upward and concave downward.
- Find the points of inflection of the graphs of functions.
- Use the Second-Derivative Test to find the relative extrema of functions.
- Analyze the graphs of functions.
- Solve real-life optimization problems.

Standards Addressed: (Learning Objectives from the AP Calculus AB Course and Exam Description)

LO 2.2A: Use derivative to analyze properties of a functions.
LO 2.3A: Interpret the meaning of a derivative within a problem.
LO 2.3B: Solve problems involving the slope of a tangent line.
LO 2.3C: Solve problems involving related rates, optimization and rectilinear motion.
LO 2.3D: Solve problems involving rates of change in applied contexts.

UNIT # 4

Unit Title: Exponential, Logarithmic, and Trigonometric Derivatives
Unit Description:

Students will now learn the rules for differentiating exponential, logarithmic and trigonometric functions. Within this unit, students will also analyze the graphs of these functions in relation to their derivatives and use derivatives to find relative extrema. This unit brings together new rules as in Unit 2 but also applications as in Unit 3 to analyze exponential, logarithmic and trigonometric functions using the skills from previous units.

LEARNING GOALS

Enduring Understanding(s):

- A function's derivative, which itself is a function, can be used to understand the behavior of the function.
- The derivative has multiple interpretations and applications including those that involve the instantaneous rates of changes.

Essential Question(s):

- How can we find derivatives of functions which the power rule does not apply to?
- How can we determine which technique to use in order to find the derivative?
- How are derivatives used to help model and analyze graphs of logarithmic and exponential functions?

Content and Skills:

Students will be able to:

- Find the derivatives of natural exponential functions.
- Use calculus to analyze the graphs of functions that involve the natural exponential function.
- Find derivatives of natural logarithmic functions.
- Use calculus to analyze the graphs of functions that involve natural logarithmic functions.
- Find derivatives of exponential and logarithmic functions involving other bases.
- Find derivatives of trigonometric functions.
- Find the relative extrema of trigonometric functions.
- Use derivatives of trigonometric functions to answer questions about real-life situations.

Standards Addressed:

LO 2.1C: Calculate derivatives.
LO 2.2A: Use derivatives to analyze properties of a function.
LO 2.3A: Interpret the meaning of a derivative within a problem.
LO 2.3B: Solve problems involving the slope of the tangent line.

UNIT # 5

Unit Title: Indefinite Integrals

Unit Description:

Students become aware of the inverse process of differentiation, antidifferentiation. They will learn techniques for integration as they relate to their corresponding derivative rules. It is critical for students to grasp the inverse relationship between deriving and antideriving, in order to help them better understand the Fundamental Theorem of Calculus in the next unit.

LEARNING GOALS

Enduring Understanding(s):
- Antidifferentiation is the inverse process of differentiation.
- The antiderivative of a function is a family of functions that vary by the constant of integration.
- Differentiation rules provide the foundation for finding antiderivatives.

Essential Question(s):
- How can we decide what technique or rules to use in order to find the antiderivative of a function?
  - What processes do we need to carry out in order to do so?
- How can we use additional information in order to determine the particular solution to an antiderivative?

Content and Skills:
Students will be able to:
- Understand the definition of antiderivative.
- Use indefinite integral notation for antiderivatives.
- Use basic integration rules to find antiderivatives.
- Use initial conditions to find particular solutions of indefinite integrals.
- Use antiderivatives to solve real-life problems.
- Use the General Power Rule to find indefinite integrals.
- Use substitution to find indefinite integrals.
- Use the General Power Rule to solve real-life problems.
- Use the Exponential Rule to find indefinite integrals (with substitution methods).
- Use the Log Rule to find indefinite integrals (with substitution methods).
- Find the six basic trigonometric integrals (without using substitution methods).

Standards Addressed: (Learning Objectives from the AP Calculus AB Course and Exam Description)
LO 3.1A: Recognize antiderivatives of basic functions.
LO 3.3B(a): Calculate antiderivatives.

UNIT #6

Unit Title: Definite Integrals
Unit Description:
Students will first learn how to approximate the area under a curve using a Riemann Sum. They will then see how to calculate the exact area under a curve by means of definite integration. Students should be able to interpret the meaning of a definite integral in context, usually as an accumulation function. Students will also calculate the area between two curves and interpret what performing that calculation means in context. It is important for students to grasp the relationship between integration and differentiation as expressed in the Fundamental Theorem of Calculus.

LEARNING GOALS

Enduring Understanding(s):

- The definite integral of a function over an interval is the limit of a Riemann sum over that interval and can be calculated using a variety of strategies.
- The Fundamental Theorem of Calculus, which has two distinct formulations, connects differentiation and integration.
- The definite integral of a function over an interval is a mathematical tool with many interpretations and applications involving accumulation.

Essential Question(s):

- How do we approximate the area underneath a curve?
- How do we find the exact area underneath a curve?
- What is the connection between integrals and derivatives?

Content and Skills:

Students will be able to:

- Evaluate definite integrals.
- Evaluate definite integrals using the Fundamental Theorem of Calculus.
- Use the Midpoint Rule and Left Endpoint Rule to approximate definite integrals.
- Use a calculator to approximate definite integrals.
- Find the area of a region bounded by the x-axis.
- Find the areas of regions bounded by two graphs.

Standards Addressed:

LO 3.2A(a): Interpret the definite integral as the limit of a Riemann sum.
LO 3.2A(b): Express the limit of a Riemann sum in integral notation.
LO 3.2B: Approximate a definite integral.
LO 3.2C: Calculate a definite integral using areas and properties of definite integrals.
LO 3.3A: Analyze functions defined by an integral.
LO 3.3B(b): Evaluate definite integrals.
LO 3.4A: Interpret the meaning of a definite integral within a problem.
LO 3.4E: Use the definite integral to solve problems in various contexts.