



Department: Mathematics

Course Name: 7th/8th Advanced

Course Description:

UNIT # 1

Unit Title: Ratios and Proportions

Unit Description: This unit will cover creating similar figures, determining whether or not two figures are similar, predicting the ratios of the lengths and areas of two similar figures, and using those relationships to scale figures up and down. This unit will develop the ability to compare quantitative information by using ratios, fractions, decimals, rates, unit rates, and percents, use comparisons to scale rates and ratios up and down, and recognize when reasoning of proportional situations is appropriate.

LEARNING GOALS

Enduring Understanding(s):

- Proportional relationships express how quantities change in relationship to each other.
- Relationships can be described and generalizations made for mathematical situations that have numbers or objects that repeat in predictable ways.
- When two quantities are compared that have the same ratio, they are proportional

Essential Question(s):

- What does it mean for two figures to be similar?
- How can you represent a relationship between two quantities?
- How can you decide what strategies to use when solving problems using fractions, decimals and percents?
- What makes a relationship proportional?

Content and Skills:

Recognize and represent proportional relationships between quantities
 Identify and use unit rates
 Use proportional relationships to solve real world problems
 Use multiple representations to identify proportions
 Apply scale factor to a variety of real world situations
 Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
 Recognize, represent and decide whether two quantities form a proportional relationship
 Use proportional relationships to solve multistep ratio and percent problems
 Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.
 Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

Standards Addressed:

CCSS.MATH.CONTENT.7.RP.A.1
 Compute unit rates associated with ratios of fractions, including ratios of lengths, areas and other quantities measured in like or different units.
 CCSS.MATH.CONTENT.7.RP.A.2
 Recognize and represent proportional relationships between quantities.
 CCSS.MATH.CONTENT.7.RP.A.2.A
 Decide whether two quantities are in a proportional relationship, e.g., by testing for equivalent ratios in a table or graphing on a coordinate plane and observing whether the graph is a straight line through the origin.
 CCSS.MATH.CONTENT.7.RP.A.2.B

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

CCSS.MATH.CONTENT.7.RP.A.2.C

Represent proportional relationships by equations.

CCSS.MATH.CONTENT.7.RP.A.2.D

Explain what a point (x, y) on the graph of a proportional relationship means in terms of the situation, with special attention to the points $(0, 0)$ and $(1, r)$ where r is the unit rate.

CCSS.MATH.CONTENT.7.RP.A.3

Use proportional relationships to solve multistep ratio and percent problems. Examples: simple interest, tax, markups and markdowns, gratuities and commissions, fees, percent increase and decrease, percent error.

CCSS.MATH.CONTENT.7.G.A.1

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

UNIT # 2

Unit Title: Linear Relationships I

Unit Description: The primary goal of this Unit is for students to develop an understanding of linear relationships. Students recognize linear relationships by the constant rate of change between two variables in a contextual situation, a table, a graph, or an equation.

LEARNING GOALS

Enduring Understanding(s):

Expressions can be manipulated to suit a particular purpose and solving problems efficiently.

Mathematical expressions, equations, and inequalities are used to model and solve real-world and mathematical problems.

The unit rate represents the slope of the related line.

Essential Question(s):

How do you differentiate between a situation that can be represented with an equation and one that can be represented with an expression?

How do variables, inequalities and algebraic expressions help you model real-world situations?

How do you decide that a situation is linear?

How do changes in one variable affect changes in a related variable?

Content and Skills:

- Use properties of operations to generate equivalent expressions.
- Solve real-life and mathematical problems using numerical and algebraic expressions and equations.
- Identify and describe the patterns of change between the independent and dependent variables for linear relationships represented by tables, graphs, equations, or contextual settings
- Construct tables, graphs, and symbolic equations that represent linear relationships
- Identify the rate of change between two variables and the y -intercept from graphs, tables, and equations that represent linear relationships
- Translate information about linear relationships given in a contextual setting, a table, a graph, or an equation to one of the other forms
- Write equations that represent linear relationships given specific pieces of information, and describe what information the variables and numbers represent

- Make a connection between slope as a ratio of vertical distance to horizontal distance between two points on a line and the rate of change between two variables that have a linear relationship
- Recognize that $y=mx$ represents a proportional relationship
- Solve problems and make decisions about linear relationships using information given in tables, graphs, and equations

Standards Addressed:

CCSS.MATH.CONTENT.7.EE.A.1

Apply properties of operations as strategies to add, subtract, factor, and expand linear expressions with rational coefficients.

CCSS.MATH.CONTENT.7.EE.A.2

Understand that rewriting an expression in different forms in a problem context can shed light on the problem and how the quantities in it are related.

CCSS.MATH.CONTENT.7.EE.B.3

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

CCSS.MATH.CONTENT.7.EE.B.4

Use variables to represent quantities in a real-world or mathematical problem, and construct simple equations and inequalities to solve problems by reasoning about the quantities.

CCSS.MATH.CONTENT.7.EE.B.4.A

Solve word problems leading to equations of the form $px + q = r$ and $p(x + q) = r$, where p , q , and r are specific rational numbers. Solve equations of these forms fluently. Compare an algebraic solution to an arithmetic solution, identifying the sequence of the operations used in each approach.

CCSS.MATH.CONTENT.7.EE.B.4.B

Solve word problems leading to inequalities of the form $px + q > r$ or $px + q < r$, where p , q , and r are specific rational numbers. Graph the solution set of the inequality and interpret it in the context of the problem.

CCSS.MATH.CONTENT.7.RP.A.2.B

Identify the constant of proportionality (unit rate) in tables, graphs, equations, diagrams, and verbal descriptions of proportional relationships.

CCSS.MATH.CONTENT.8.EE.B.5

Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

CCSS.MATH.CONTENT.8.F.A.2

Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in

tables, or by verbal descriptions).

CCSS.MATH.CONTENT.8.F.A.3

Interpret the equation $y = mx + b$ as defining a linear function, whose graph is a straight line; give examples of functions that are not linear.

CCSS.MATH.CONTENT.8.F.B.4

Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

UNIT # 3

Unit Title: Exponents, Radicals & The Pythagorean Theorem

Unit Description: This unit explores the Pythagorean Theorem, square roots, cube roots, and strategies for estimating square and cube roots. The set of real numbers is extended from only rational numbers to also include irrational numbers.

LEARNING GOALS

Enduring Understanding(s):

- Every number has a decimal expansion, for rational numbers it repeats eventually
- Knowing the properties of integer exponents is a more efficient way to simplify exponential expressions.
- Scientific notation is useful when calculating with very large or very small quantities and to compare quantities.
- The Pythagorean Theorem models the special relationship among the side lengths of right triangles.
- Exponents and roots are inverse operations

Essential Question(s):

- How does knowing the Pythagorean Theorem help me in real world situations?
- How do I determine the best numerical representation for a given situation?
- How does knowing the properties of exponents help me simplify expressions?
- Why is it useful for me to know the root of a number?

Content and Skills:

- Know that there are numbers that are not rational, and approximate them by rational numbers.
- Write, interpret, and operate with numerical expressions in scientific notation
- Write and interpret equivalent expressions using the rules for exponents and operations
- Solve problems that involve exponents, including scientific notation
- Develop strategies for finding the distance between two points on a coordinate grid
- Explain a proof of the Pythagorean Theorem

- Use the Pythagorean Theorem and its converse to solve a variety of problems
- Estimate the value of square and cube roots
- Solve square roots that are both rational and irrational
- Compare numbers that can be represented as fractions (rational numbers) to numbers that cannot be represented as fractions (irrational numbers) and recognize that the set of real numbers consists of rational and irrational numbers

Standards Addressed:

CCSS.MATH.CONTENT.7.NS.A.2.D

Convert a rational number to a decimal using long division; know that the decimal form of a rational number terminates in 0s or eventually repeats.

CCSS.MATH.CONTENT.8.NS.A.1

Know that numbers that are not rational are called irrational. Understand informally that every number has a decimal expansion; for rational numbers show that the decimal expansion repeats eventually, and convert a decimal expansion which repeats eventually into a rational number.

CCSS.MATH.CONTENT.8.NS.A.2

Use rational approximations of irrational numbers to compare the size of irrational numbers, locate them approximately on a number line diagram, and estimate the value of expressions (e.g., π^2).

CCSS.MATH.CONTENT.8.EE.A.1

Know and apply the properties of integer exponents to generate equivalent numerical expressions. For example, $32 \times 3^{-5} = 3^{-3} = 1/3^3 = 1/27$.

CCSS.MATH.CONTENT.8.EE.A.2

Use square root and cube root symbols to represent solutions to equations of the form $x^2 = p$ and $x^3 = p$, where p is a positive rational number. Evaluate square roots of small perfect squares and cube roots of small perfect cubes. Know that $\sqrt{2}$ is irrational.

CCSS.MATH.CONTENT.8.EE.A.3

Use numbers expressed in the form of a single digit times an integer power of 10 to estimate very large or very small quantities, and to express how many times as much one is than the other.

CCSS.MATH.CONTENT.8.EE.A.4

Perform operations with numbers expressed in scientific notation, including problems where both decimal and scientific notation are used. Use scientific notation and choose units of appropriate size for measurements of very large or very small quantities (e.g., use millimeters per year for seafloor spreading). Interpret scientific notation that has been generated by technology

CCSS.MATH.CONTENT.8.G.B.6

Explain a proof of the Pythagorean Theorem and its converse.

CCSS.MATH.CONTENT.8.G.B.7

Apply the Pythagorean Theorem to determine unknown side lengths in right triangles in real-world and mathematical

problems in two and three dimensions.

CCSS.MATH.CONTENT.8.G.B.8

Apply the Pythagorean Theorem to find the distance between two points in a coordinate system.

UNIT #4

Unit Title: 2D Geometry

Unit Description: This unit develops the understanding of congruence and similarity. Parallel lines, transversals, and transformations are also in this unit.

LEARNING GOALS

Enduring Understanding(s):

- Geometric figures can change size and/or position while maintaining proportional attributes.

Essential Question(s):

- How can you use coordinates and algebraic techniques to represent, interpret and verify geometric relationships?
- How do geometric models describe spatial relationships?

Content and Skills:

Recognize properties of reflection, rotation, and translation transformations

Explore techniques for using rigid motion transformations to create symmetric designs

Use coordinate rules for basic rigid motion transformations

Recognize that two figures are congruent if one is derived from the other by a sequence of reflection, rotation, and/or translation transformations

Recognize that two figures are similar if one can be obtained from the other by a sequence of reflections, rotations, translations, and/or dilations

Use transformations to describe a sequence that exhibits the congruence between figures

Relate properties of angles formed by parallel lines and transversals, and the angle sum in any triangle, to properties of transformations

Use properties of congruent and similar triangles to solve problems about shapes and measurements

CCSS.MATH.CONTENT.8.G.A.1

Verify experimentally the properties of rotations, reflections, and translations:

CCSS.MATH.CONTENT.8.G.A.1.A

Lines are taken to lines, and line segments to line segments of the same length.

CCSS.MATH.CONTENT.8.G.A.1.B

Angles are taken to angles of the same measure.

CCSS.MATH.CONTENT.8.G.A.1.C

Parallel lines are taken to parallel lines.

CCSS.MATH.CONTENT.8.G.A.2

Understand that a two-dimensional figure is congruent to another if the second can be obtained from the first by a sequence of rotations, reflections, and translations; given two congruent figures, describe a sequence that exhibits the congruence between them.

CCSS.MATH.CONTENT.8.G.A.3

Describe the effect of dilations, translations, rotations, and reflections on two-dimensional figures using coordinates.

CCSS.MATH.CONTENT.8.G.A.4

Understand that a two-dimensional figure is similar to another if the second can be obtained from the first by a sequence of rotations, reflections, translations, and dilations; given two similar two-dimensional figures, describe a sequence that exhibits the similarity between them.

CCSS.MATH.CONTENT.8.G.A.5

Use informal arguments to establish facts about the angle sum and exterior angle of triangles, about the angles created when parallel lines are cut by a transversal, and the angle-angle criterion for similarity of triangles.

UNIT # 5

Unit Title: 3D Geometry

Unit Description: This unit focuses on developing strategies for measuring surface area and volume of prisms and cylinders, including area and circumference of circles. Cones and spheres are also investigated to develop volume relationships between the 3D shapes.

LEARNING GOALS

Enduring Understanding(s):

- Geometric relationships exist between two-dimensional and three dimensional figures.
- “Pi” (π) is the relationship between a circle’s circumference and diameter.
- Volume measures capacity, surface area measures coverings

Essential Question(s):

- How does the shape of an object influence decisions when determining volume and surface area?
- How can you design a package to hold maximum volume with minimal surface area?
- How can one part of a circle help determine the measure of another part?
- How does scaling an object affect its volume and surface area?

Content and Skills:

- Visualize three-dimensional shapes and the effects of slicing those shapes by planes
- Predict the effects of scaling dimensions on linear, surface area, and volume measures of prisms, cylinders, and other figures
- Investigate the relationship between volumes of prisms and volumes of cylinders as well as the relationship between surface areas of prisms and surface areas of cylinders
- Solve problems involving surface areas and volumes of solid figures
- Discover that volumes of prisms and cylinders can be calculated as the product of the area of the base and the height

Standards Addressed:

CCSS.MATH.CONTENT.7.EE.B.3

Solve multi-step real-life and mathematical problems posed with positive and negative rational numbers in any form (whole numbers, fractions, and decimals), using tools strategically. Apply properties of operations to calculate with numbers in any form; convert between forms as appropriate; and assess the reasonableness of answers using mental computation and estimation strategies.

CCSS.MATH.CONTENT.7.NS.A.2.C

Apply properties of operations as strategies to multiply and divide rational numbers.

CCSS.MATH.CONTENT.7.RP.A.2

Recognize and represent proportional relationships between quantities.

CCSS.MATH.CONTENT.7.NS.A.3

Solve real-world and mathematical problems involving the four operations with rational numbers.

CCSS.MATH.CONTENT.7.G.A.1

Solve problems involving scale drawings of geometric figures, including computing actual lengths and areas from a scale drawing and reproducing a scale drawing at a different scale.

CCSS.MATH.CONTENT.7.G.A.3

Describe the two-dimensional figures that result from slicing three-dimensional figures, as in plane sections of right rectangular prisms and right rectangular pyramids.

CCSS.MATH.CONTENT.7.G.B.4

Know the formulas for the area and circumference of a circle and use them to solve problems; give an informal derivation of the relationship between the circumference and area of a circle.

CCSS.MATH.CONTENT.7.G.B.6

Solve real-world and mathematical problems involving area, volume and surface area of two- and three-dimensional objects composed of triangles, quadrilaterals, polygons, cubes, and right prisms.

CCSS.MATH.CONTENT.8.G.C.9

Know the formulas for the volumes of cones, cylinders, and spheres and use them to solve real-world and mathematical problems.

UNIT #6

Unit Title: Probability

Unit Description: Students will gain an understanding of experimental and theoretical probabilities and the relationship between them. The Unit also makes important connections between probability and rational numbers, geometry, statistics, science, and business.

LEARNING GOALS

Enduring Understanding(s):

- Knowing all possible outcomes and their likelihoods helps to make informed decisions.
- The relationship among events affects probability.

Essential Question(s):

- Does the outcome of one event have an impact on the outcome of subsequent events?
- How do you describe the probability of events?
- What does it mean for a probability to be 0 or 1?
- Why do the results of experiments sometimes differ from the theoretical probability?
- How can I use probability to help me predict what can happen in real life?

Content and Skills:

- Recognize that probabilities are useful for predicting what will happen over the long run
- For an event described in everyday language, identify the outcomes in a sample space that compose the event
- Interpret experimental and theoretical probabilities and the relationship between them and recognize that experimental probabilities are better estimates of theoretical probabilities when they are based on larger numbers
- Distinguish between outcomes that are equally likely or not equally likely by collecting data and analyzing experimental probabilities
- Realize that the probability of simple events is a ratio of favorable outcomes to all outcomes in the sample space
- Recognize that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of

the event occurring

- Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability
- Determine the fairness of a game

Standards Addressed:

CCSS.MATH.CONTENT.7.SP.A.1

Understand that statistics can be used to gain information about a population by examining a sample of the population; generalizations about a population from a sample are valid only if the sample is representative of that population. Understand that random sampling tends to produce representative samples and support valid inferences.

CCSS.MATH.CONTENT.7.SP.A.2

Use data from a random sample to draw inferences about a population with an unknown characteristic of interest. Generate multiple samples (or simulated samples) of the same size to gauge the variation in estimates or predictions.

CCSS.MATH.CONTENT.7.SP.B.3

Informally assess the degree of visual overlap of two numerical data distributions with similar variabilities, measuring the difference between the centers by expressing it as a multiple of a measure of variability.

CCSS.MATH.CONTENT.7.SP.B.4

Use measures of center and measures of variability for numerical data from random samples to draw informal comparative inferences about two populations.

CCSS.MATH.CONTENT.7.SP.C.5

Understand that the probability of a chance event is a number between 0 and 1 that expresses the likelihood of the event occurring. Larger numbers indicate greater likelihood. A probability near 0 indicates an unlikely event, a probability around $1/2$ indicates an event that is neither unlikely nor likely, and a probability near 1 indicates a likely event.

CCSS.MATH.CONTENT.7.SP.C.6

Approximate the probability of a chance event by collecting data on the chance process that produces it and observing its long-run relative frequency, and predict the approximate relative frequency given the probability.

CCSS.MATH.CONTENT.7.SP.C.7

Develop a probability model and use it to find probabilities of events. Compare probabilities from a model to observed frequencies; if the agreement is not good, explain possible sources of the discrepancy.

CCSS.MATH.CONTENT.7.SP.C.7.A

Develop a uniform probability model by assigning equal probability to all outcomes, and use the model to determine probabilities of events.

CCSS.MATH.CONTENT.7.SP.C.7.B

Develop a probability model (which may not be uniform) by observing frequencies in data generated from a chance process.

CCSS.MATH.CONTENT.7.SP.C.8

Find probabilities of compound events using organized lists, tables, tree diagrams, and simulation.

CCSS.MATH.CONTENT.7.SP.C.8.A

Understand that, just as with simple events, the probability of a compound event is the fraction of outcomes in the sample space for which the compound event occurs.

CCSS.MATH.CONTENT.7.SP.C.8.B

Represent sample spaces for compound events using methods such as organized lists, tables and tree diagrams. For an event described in everyday language (e.g., "rolling double sixes"), identify the outcomes in the sample space which compose the event.

CCSS.MATH.CONTENT.7.SP.C.8.C

Design and use a simulation to generate frequencies for compound events.

UNIT # 7

Unit Title: Linear Relationships Part 2

Unit Description: This unit explores situations that can be represented with various mathematical models, including graphs and equations. Variability and correlation between two variables is also discussed. Students also develop an understanding of the ways in which systems of equations and inequalities can be used to model problem situations.

LEARNING GOALS

Enduring Understanding(s):

- Linear functions represent situations involving a constant rate of change.
- The characteristics of linear functions and their representations are useful in solving real world problems.
- Systems of linear equations can be used to model many types of real world problems.

Essential Question(s):

- What do the solutions of a system of linear equations represent?
- What are the advantages and disadvantages of solving a system of linear equations graphically and algebraically?
- How can systems of equations be used to represent situations and solve problems?
- How can you determine if your line of best fit is representative of the given data?
- What do you need to consider when solving a system of equations?

Content and Skills:

Use data to make predictions

Fit a line to data that show a linear trend and measure closeness of fit

Analyze scatter plots of bivariate data to determine the strength of the linear association between the two variables

Use correlation coefficients informally to describe the strength of the linear association illustrated by scatter plots

Recognize that the form $Ax+By=C$ of linear equations is equivalent to the form $y=mx+b$ for linear equations

Recognize that solving a system of linear equations is equivalent to finding values of the variables that will simultaneously satisfy all equations in the system

Develop skills in solving systems of linear equations by graphing solutions of separate equations; writing the system of equations in equivalent $y=mx+b$ form; or using combinations of the system to eliminate one variable

Choose between graphing and symbolic methods to efficiently find the solution to a particular system of linear equations

Gain fluency with symbol manipulation in solving systems of linear equations

Solve problems that involve systems of linear equations

Continue to develop skill in solving a linear inequality in two variables by graphing and symbolic methods

Develop skill in solving systems of linear inequalities by graphing solutions of each inequality and finding the region of feasible points that satisfy both inequalities; and solving inequalities to find pairs of numbers that satisfy both inequalities

Choose between graphing and symbolic methods to efficiently find the region of feasible points to a particular system of linear inequalities

Solve a simple system consisting of a linear equation and a quadratic equation in two variables symbolically and graphically

Solve problems that involve linear inequalities or systems of linear inequalities

Standards Addressed:

8.EE.B.5 Graph proportional relationships, interpreting the unit rate as the slope of the graph. Compare two different proportional relationships represented in different ways. For example, compare a distance-time graph to a distance-time equation to determine which of two moving objects has greater speed.

8.EE.C.7 Solve linear equations in one variable.

8.EE.C.7b Solve linear equations with rational number coefficients, including equations whose solutions require expanding expressions using the distributive property and collecting like terms.

8.EE.C.8 Analyze and solve pairs of simultaneous linear equations.

8.EE.C.8a Understand that solutions to a system of two linear equations in two variables correspond to points of intersection of their graphs, because points of intersection satisfy both equations simultaneously.

8.EE.C.8b Solve systems of two linear equations in two variables algebraically, and estimate solutions by graphing the equations. Solve simple cases by inspection.

8.EE.C.8c Solve real-world and mathematical problems leading to two linear equations in two variables.

8.F.A.1 Understand that a function is a rule that assigns to each input exactly one output. The graph of a function is the set of ordered pairs consisting of an input and the corresponding output.

8.F.A.2 Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions). For example, given a linear function represented by a table of values and a linear function represented by an algebraic expression, determine which function has the greater rate of change.

8.F.A.3 Interpret the equation $y=mx+b$ as defining a linear function whose graph is a straight line; give examples of functions that are not linear.

8.F.B.4 Construct a function to model a linear relationship between two quantities. Determine the rate of change and initial value of the function from a description of a relationship or from two (x, y) values, including reading these from a table or from a graph. Interpret the rate of change and initial value of a linear function in terms of the situation it models, and in terms of its graph or a table of values.

8.F.B.5 Describe qualitatively the functional relationship between two quantities by analyzing a graph (e.g., where the function is increasing or decreasing, linear or nonlinear). Sketch a graph that exhibits the qualitative features of a function that has been described verbally.

8.SP.A.1 Construct and interpret scatter plots for bivariate measurement data to investigate patterns of association between two quantities. Describe patterns such as clustering, outliers, positive or negative association, linear association, and nonlinear association.

8.SP.A.2 Know that straight lines are widely used to model relationships between two quantitative variables. For scatter plots that suggest a linear association, informally fit a straight line, and informally assess the model fit by judging the closeness of the data points to the line.

8.SP.A.3 Use the equation of a linear model to solve problems in the context of bivariate measurement data, interpreting the slope and intercept.

8.SP.A.4 Understand that patterns of association can also be seen in bivariate categorical data by displaying frequencies and relative frequencies in a two-way table. Construct and interpret a two-way table summarizing data on two categorical variables collected from the same subjects. Use relative frequencies calculated for rows or columns to describe possible association between the two variables.