Milford Public Schools Curriculum

Department: Mathematics
Course Name: Precalculus Level 1

UNIT 1

Unit Title:
Unit Description: Students will construct polynomial graphs with zeros and end behavior, and apply limit notation to the end behavior of functions. The greater focus in this course lies in understanding and applying the Fundamental Theorem of Algebra, Rational Root Theorem, Remainder Theorem, and Upper/Lower Bound Tests and using them to solve higher degree equations. Students will focus on dividing rational functions and graphing rational equations using intercepts and end behavior around horizontal and vertical asymptotes. Students will use limit notation to describe behavior around vertical asymptotes.

LEARNING GOALS

Enduring Understanding(s):

- The field of complex numbers is algebraically closed.
- All polynomials with real coefficients of degree *n* have exactly *n* complex roots, counting multiplicity.
- Analysis of rational functions requires deconstruction of the underlying polynomial functions.

Essential Question(s):

- What connections exist between the roots of a polynomial function and the features of its graph?
- How does the equation of a polynomial or rational function provide information about its graph and viceversa?
- How can we know the type of roots that a polynomial function has?
- How can we decide what function to use in order to model real-world situations? What processes do we need to carry out in order to do so?

Content and Skills:

- Identify Polynomial Functions and Their Degree
- Graph Polynomial Functions Using Transformations
- Identify the Real Zeros of a Polynomial Functions and Their Multiplicity
- Analyze the Graph of a Polynomial Function
- Build Cubic Models from Data
- Use the Remainder and Factor Theorems
- Use the Rational Zeros Theorem to List the Potential Rational Zeros of a Polynomial Function
- Find the Real Zeros of a Polynomial Function
- Solve Polynomial Equations
- Use the Theorem for Bounds on Zeros
- Use the Intermediate Value Theorem
- Use the Conjugate Pairs Theorem
- Find a Polynomial Functions with Specified Zeros
- Find the Complex Zeros of a Polynomial Function
- Find the Domain of a Rational Function
- Find the Vertical Asymptotes of a Rational Function
- Find the Horizontal or Oblique Asymptotes of a Rational Function
- Analyze the Graph of a Rational Function
- Solve Applied Problems Involving Rational Functions
- Solve Polynomial Inequalities Algebraically and Graphically *(*supplemental)*
- Solve Rational Inequalities Algebraically and Graphically *(*supplemental)*

Standards Addressed:
CCSS.MATH.CONTENT.HSA.APR.B.2
Know and apply the Remainder Theorem: For a polynomial $p(x)$ and a number a, the remainder on division by x a
Is $p(a)$, so $p(a) = 0$ if and only if $(x$ $a)$
is a factor of $p(x)$.
CCSS.MATH.CONTENT.HSA.APR.B.3
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial.
CCSS.MATH.CONTENT.HSF.IF.C.7.C
Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
CCSS.MATH.CONTENT.HSF.IF.C.7.D
(+) Graph rational functions, identifying zeros and asymptotes when suitable factorizations are available, and showing end behavior.

UNIT 2

Unit Title:
Unit Description: While students gained basic knowledge of the properties of logarithms and exponents in Algebra 2, they will expand their understanding to include a strong working facility with exponential and logarithmic functions. Students will also work further with the properties of composite functions, one-to-one functions, and inverse functions.

LEARNING GOALS

Enduring Understanding(s):

- Properties of logarithmic and exponential functions are related.
- Inverse relationships exist between logarithms and exponents.

Essential Question(s):

- How can we decide what function to use in order to model real-world situations? What processes do we need to carry out in order to do so?
- What purposes do logarithms serve that no other function can accomplish?

Content and Skills:
Students will be able to:

- Form a Composite Function
- Find the Domain of a Composite Function
- Determine Whether a Function is One-to-One
- Determine the Inverse of a Function Defined by a Map or a Set of Ordered Pairs
- Obtain the Graph of the Inverse Functions from the Graph of the Function
- Find the Inverse of a Functions Defined by an Equation
- Evaluate Exponential Functions
- Graph Exponential Functions
- Define the Number *e*
- Solve Exponential Equations
- Change Exponential Statements to Logarithmic Statements and Logarithmic Statements to Exponential Statements
- Evaluate Logarithmic Expressions
- Determine the Domain of a Logarithmic Function
- Graph Logarithmic Functions

- Solve Logarithmic Equations
- Work with the Properties of Logarithms
- Write a Logarithmic Expression as a Sum or Difference of Logarithms
- Write a Logarithmic Expression as a Single Logarithm
- Evaluate Logarithms Whose Base Is Neither 10 Nor *e*
- Graph Logarithmic Functions Whose Base Is Neither 10 Nor *e*
- Solve Logarithmic and Exponential Equations Using a Graphing Utility

Standards Addressed:
CCSS.MATH.CONTENT.HSF.BF.A.1
Write a function that describes a relationship between two quantities.*
CCSS.MATH.CONTENT.HSF.BF.A.1.A
Determine an explicit expression, a recursive process, or steps for calculation from a context.
CCSS.MATH.CONTENT.HSF.BF.A.1.B
Combine standard function types using arithmetic operations. *For example, build a function that models the temperature of a cooling body by adding a constant function to a decaying exponential, and relate these functions to the model.*
CCSS.MATH.CONTENT.HSF.BF.A.1.C
(+) Compose functions. *For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.*
CCSS.MATH.CONTENT.HSF.IF.C.7.E
Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
CCSS.MATH.CONTENT.HSF.BF.B.3
Identify the effect on the graph of replacing $f(x)$ by $f(x) + k$, $k\,f(x)$, $f(kx)$, and $f(x + k)$ for specific values of k (both positive and negative); find the value of k given the graphs. Experiment with cases and illustrate an explanation of the effects on the graph using technology. Include recognizing even and odd functions from their graphs and algebraic expressions for them.
CCSS.MATH.CONTENT.HSF.BF.B.4
Find inverse functions.
CCSS.MATH.CONTENT.HSF.BF.B.4.A
Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. *For example, f(x) =2 x3 or f(x) = (x+1)/(x1)*
for x ≠ 1.
CCSS.MATH.CONTENT.HSF.BF.B.4.B
(+) Verify by composition that one function is the inverse of another.
CCSS.MATH.CONTENT.HSF.BF.B.4.C
(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.
CCSS.MATH.CONTENT.HSF.BF.B.5
(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

UNIT 3

Unit Title:
Unit Description: Students will measure angles using both degrees and radians, while applying this understanding to the six trigonometric ratios of any given angle. The will then use their knowledge of the functions' domain and range to construct graphs of the six trigonometric functions and their inverses.

LEARNING GOALS

Enduring Understanding(s):	**Essential Question(s):**
• Angles are the domain elements of the six trigonometric functions. • Understanding reference angles can help find solutions to trigonometric ratios in all quadrants. • The graphs of sine, cosine, and tangent and their inverses are unique because they are periodic functions with continuous subintervals.	• What determines when radians or degrees are the more useful measure of an angle? • What information can be gained about the equation of a trigonometric function from its graph and vice versa? • How can a trigonometric function be evaluated for angles beyond the domain of quadrant I?

Content and Skills:
Students will be able to:
● Convert between decimal and degree, minute, second measures for angles
● Find the length of an arc of a circle
● Convert from degrees to radians and from radians to degrees
● Find the area of a sector of a circle
● Find the linear speed of an object traveling in circular motion
● Find the exact values of the trigonometric functions using a point on the unit circle
● Find the exact values of the trigonometric functions of quadrantal angles
● Use a calculator to approximate the value of a trigonometric function
● Use a circle of radius *r* to evaluate the trigonometric functions
● Determine the domain and range of the trigonometric functions
● Determine the period of the trigonometric functions
● Determine the signs of the trigonometric functions in a given quadrant
● Find the values of the trigonometric functions using fundamental identities
● Find the exact values of the trigonometric functions of an angle given one of the functions and the quadrant of the angle
● Use even-odd properties to find the exact values of the trigonometric functions
● Graph functions of the form *y* = *A* sin () using transformations
● Graph functions of the form *y* = *A* cos () using transformations
● Determine the amplitude and period of sinusoidal functions
● Graph sinusoidal functions using key points
● Find an equation for a sinusoidal graph
● Graph functions of the form *y* = *A* tan (+ *B* and *y* = *A* cot () + *B*
● Graph functions of the form *y* = *A* csc (+ *B* and *y* = *A* sec () + *B*
● Graph sinusoidal functions of the form *y* = *A* sin () + *B*
● Build sinusoidal models from data
● Find the exact values of an inverse sine function
● Find an approximate value of an inverse sine function
● Use properties of inverse functions to find the exact values of certain composite functions
● Find the inverse function of a trigonometric function
● Solve equations involving inverse trigonometric expression as an algebraic expression
● Find the exact value of expressions involving the inverse sine, cosine, and tangent functions
● Define the inverse secant, cosecant, and cotangent functions
● Use a calculator to evaluate inverse sec, csc, and cot
● Write a trigonometric expression as an algebraic expression

Standards Addressed:
CCSS.MATH.CONTENT.HSF.TF.A.1 Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
CCSS.MATH.CONTENT.HSF.TF.A.2 Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around

the unit circle.
CCSS.MATH.CONTENT.HSF.TF.A.3 Use special triangles to determine geometrically the values of sine, cosine, tangent for π/3, π/4 and π/6, and use the unit circle to express the values of sine, cosine, and tangent for *x*, π + *x*, and 2π *x* *x* in terms of their values for *x*, where *x* is any real number.
CCSS.MATH.CONTENT.HSF.TF.A.4 Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions. Model periodic phenomena with trigonometric functions.
CCSS.MATH.CONTENT.HSF.TF.B.5 Choose trigonometric functions to model periodic phenomena with specified amplitude, frequency, and midline.*
CCSS.MATH.CONTENT.HSF.TF.B.6 Understand that restricting a trigonometric function to a domain on which it is always increasing or always decreasing allows its inverse to be constructed.
CCSS.MATH.CONTENT.HSF.TF.B.7 Use inverse functions to solve trigonometric equations that arise in modeling contexts; evaluate the solutions using technology, and interpret them in terms of the context.*
CCSS.MATH.CONTENT.HSF.BF.B.4.D Produce an invertible function from a noninvertible function by restricting the domain.

UNIT 4

Unit Title:
Unit Description: In Geometry, students were exposed to solving right and oblique triangles. Students will continue to work with the trigonometric ratios, the Law of Sines and Cosines, and the area formulas for triangles. Students will also expand their work of proofs and utilization of trigonometric identities to include the fundamental, reciprocal, cofunction, odd/even, sum/difference, and multiple angle identities.

LEARNING GOALS

Enduring Understanding(s):

- Right triangles are used to solve real-world triangles.
- Identities are used to evaluate, simplify, and solve trigonometric expressions and equations.

Essential Question(s):

- How can we decide what trigonometric relationships to use in order to solve both right and oblique triangles?
- How can we find and prove various trigonometric identities and formulas?
- How can we decide what function to use in order to model real-world situations?
- What processes do we need to carry out in order to do so?

Content and Skills:
Students will be able to:

- Solve equations involving a single trigonometric function
- Solve trigonometric equations using a calculator
- Solve trigonometric equations quadratic in form
- Solve trigonometric equations using fundamental identities
- Solve trigonometric equations using a graphing utility
- Use algebra to simplify trigonometric expressions
- Establish identities
- Use sum and difference formulas to find exact values
- Use sum and difference formulas to establish identities
- Use sum and difference formulas involving inverse trigonometric functions
- Solve trigonometric equations linear in sine and cosine
- Use double-angle formulas to find exact values
- Use double-angle formulas to establish identities

- Use half-angle formulas to find exact values
- Express products as sums
- Express sum as products
- Find the Value of Trigonometric Functions of Acute Angles Using Right Triangles
- Use the Complementary Angle Theorem
- Solve Right Triangles
- Solve Applied Problems
- Solve SAA or ASA Triangles
- Solve SSA Triangles
- Solve SAS Triangles
- Solve SSS Triangles
- Find the Area of SAS Triangles
- Find the Area of SSS Triangles
- Build a Model for an Object in Simple Harmonic Motion
- Analyze an Object in Damped Motion
- Graph the Sum of Two Functions

Standards Addressed:
CCSS.MATH.CONTENT.HSF.TF.C.8
Prove the Pythagorean identity sin2(θ) + cos2(θ) = 1 and use it to find sin(θ), cos(θ), or tan(θ) given sin(θ), cos(θ), or tan(θ) and the quadrant of the angle.
CCSS.MATH.CONTENT.HSF.TF.C.9
(+) Prove the addition and subtraction formulas for sine, cosine, and tangent and use them to solve problems.
CCSS.MATH.CONTENT.HSG.SRT.D.9
(+) Derive the formula $A = 1/2\, ab \sin(C)$ for the area of a triangle by drawing an auxiliary line from a vertex perpendicular to the opposite side.
CCSS.MATH.CONTENT.HSG.SRT.D.10
(+) Prove the Laws of Sines and Cosines and use them to solve problems.
CCSS.MATH.CONTENT.HSG.SRT.D.11
(+) Understand and apply the Law of Sines and the Law of Cosines to find unknown measurements in right and nonright triangles (e.g., surveying problems, resultant forces).

UNIT 5

Unit Title:
Unit Description: Students will develop an understanding of vectors and their applications to real-world problems. Students will explore how parametric equations can be used to model real-world scenarios. Finally, students will learn how to convert between rectangular and polar coordinates, describe when each coordinate system is most useful, as well as describe the graph of polar functions.

LEARNING GOALS

Enduring Understanding(s):
- Every point in a two-dimensional plane can be represented by either a unique rectangular or polar ordered pair.
- Vector analysis has multiple applications in science, especially in the area of physics.
- Parametric equations allow two-dimensional
- motion to be expressed with respect to a third variable, often used in physics to depict motion as a

Essential Question(s):
- How do multiple coordinate systems better help us to model the world around us?

function of time.	

Content and Skills:
Students will be able to:

- Plot points using polar coordinates
- Convert from polar coordinates to rectangular coordinates
- Convert from rectangular coordinates to polar coordinates
- Transform equations between polar and rectangular forms
- Identify and graph polar equations by converting to rectangular equations
- Graph polar equations using a graphing utility
- Test polar equations for symmetry
- Graph polar equations by plotting points
- Plot points in the complex plane
- Convert a complex number between rectangular form and polar form
- Find products and quotients of complex numbers in polar form
- Use De Moivre's Theorem
- Find complex roots
- Graph vectors
- Find a position vector
- Add and subtract vectors algebraically
- Find a scalar multiple and the magnitude of a vector
- Find a unit vector
- Find a vector from its direction and magnitude
- Model with vectors
- Graph parametric equations by hand
- Graph parametric equations using a graphing utility
- Find a rectangular equation for curve defined parametrically
- Use time as a parameter in parametric equations
- Find parametric equations for curves defined by rectangular equations
- Find the dot product of two vectors *(*supplemental)*
- Find the angle between two vectors *(*supplemental)*
- Determine whether two vectors are parallel *(*supplemental)*
- Determine whether two vectors are orthogonal *(*supplemental)*
- Decompose a vector into two orthogonal vectors *(*supplemental)*
- Find the distance between two points in space *(*supplemental)*
- Find the position vectors in space *(*supplemental)*
- Perform operations on vectors *(*supplemental)*
- Find the cross product of two vectors *(*supplemental)*
- Know algebraic properties of the cross product *(*supplemental)*
- Know geometric properties of the cross product *(*supplemental)*
- Find a vector orthogonal to two given vectors *(*supplemental)*
- Find the area of parallelogram *(*supplemental)*

Standards Addressed:
CCSS.MATH.CONTENT.CN.B.4
(+) Represent complex numbers on the complex plane in rectangular and polar form (including real and imaginary numbers), and explain why the rectangular and polar forms of a given complex number represent the same number.
Represent and model with vector quantities.
CCSS.MATH.CONTENT.HSN.VM.A.1
(+) Recognize vector quantities as having both magnitude and direction. Represent vector quantities by directed line segments, and use appropriate symbols for vectors and their magnitudes (e.g., $\boldsymbol{v}$, $|\boldsymbol{v}|$, $||\boldsymbol{v}||$, v).
CCSS.MATH.CONTENT.HSN.VM.A.2
(+) Find the components of a vector by subtracting the coordinates of an initial point from the coordinates of a terminal

point.
CCSS.MATH.CONTENT.HSN.VM.A.3
(+) Solve problems involving velocity and other quantities that can be represented by vectors. Perform operations on vectors.
CCSS.MATH.CONTENT.HSN.VM.B.4
(+) Add and subtract vectors.
CCSS.MATH.CONTENT.HSN.VM.B.4.A
Add vectors end-to-end, component-wise, and by the parallelogram rule. Understand that the magnitude of a sum of two vectors is typically not the sum of the magnitudes.
CCSS.MATH.CONTENT.HSN.VM.B.4.B
Given two vectors in magnitude and direction form, determine the magnitude and direction of their sum.
CCSS.MATH.CONTENT.HSN.VM.B.4.C
Understand vector subtraction ***v w*** as ***v*** + (***w***), where ***w*** is the additive inverse of ***w***, with the same magnitude as ***w*** and pointing in the opposite direction. Represent vector subtraction graphically by connecting the tips in the appropriate order, and perform vector subtraction component-wise.
CCSS.MATH.CONTENT.HSN.VM.B.5
(+) Multiply a vector by a scalar.
CCSS.MATH.CONTENT.HSN.VM.B.5.A
Represent scalar multiplication graphically by scaling vectors and possibly reversing their direction; perform scalar multiplication component-wise, e.g., as $c(vx, vy) = (cvx,cvy)$.
CCSS.MATH.CONTENT.HSN.VM.B.5.B
Compute the magnitude of a scalar multiple $c\mathbf{v}$ using $||c\mathbf{v}|| = |c|\mathbf{v}$. Compute the direction of $c\mathbf{v}$ knowing that when $|c|\mathbf{v} \neq 0$, the direction of $c\mathbf{v}$ is either along $\mathbf{v}$ (for $c > 0$) or against $\mathbf{v}$ (for $c < 0$).

UNIT 6

Unit Title:
Unit Description: The students will learn about sequences and extend students' knowledge to include arithmetic and geometric series, both finite and infinite. Summation notation and properties of sums are also introduced. This unit will define what limit means and how to calculate it. Limits will be found graphically, algebraically, and using a calculator. The unit also explains how limits are used to test for continuity and finally how they will relate to finding the slope of the tangent line.

LEARNING GOALS

Enduring Understanding(s):

- A sequence of numbers can often be represented by a function where the domain is the set of natural numbers.
- Limits support the concepts of convergence and continuity.

Essential Question(s):

- How can an infinite pattern of numbers or their sum be expressed in a simpler algebraic form?
- How can we model the behavior of a function as it approaches a certain input value versus the behavior *at* that value?

Content and Skills:
Students will be able to:
● Write the first several terms of a sequence
● Write the terms of a sequence defined by a recursive formula
● Use Summation Notation
● Find the sum of a sequence algebraically and graphing
● Determine whether a sequence is arithmetic
● Find a formula for an arithmetic sequence.
● Determine whether a sequence is geometric

- Find a formula for a geometric sequence
- Find the sum of a geometric sequence
- Find a limit using a table
- Find a limit using a graph
- Find the limit of a sum, a difference, and a product
- Find the limit of a polynomial
- Find the limit of a power or a root
- Find the limit of a quotient
- Find the limit of an average rate of change
- Find the one-sided limits of functions

Standards Addressed: