Milford Public Schools Curriculum

Department: Mathematics
Course Name: Geometry (L1 & L2)

UNIT 1

Unit Title: Connecting Algebra and Geometry Through Coordinates
Unit Description:

LEARNING GOALS

Enduring Understanding(s):

- The correspondence between numerical coordinates and geometric points allows methods from algebra to be applied to geometry and vice versa.
- Algebraic formulas can be used to determine and verify geometric relationships between and among parts of geometric figures on a coordinate plane.
- Coordinate Geometry connects algebra and geometry, resulting in powerful methods of analysis and problem solving.

Essential Question(s):

- How is the coordinate plane useful in solving Geometric problems?
- How can we use the properties of Algebra to verify the properties of Geometric figures on a coordinate plane?

Content and Skills:

- Define basic geometric terms.
- Write an equation of a parallel line given two points, a point and a slope, or an equation and a point.
- Write an equation of a perpendicular line given two points, a point and a slope, or an equation and a point.
- Find the perimeter of a polygon on a coordinate plane using the distance formula or pythagorean theorem.
- Find the area of a polygon by sectioning and utilizing the surrounding area.
- Find the distance between two points.
- Find the midpoint between two points.
- Find a second endpoint given a midpoint and endpoint.
- Enclose a region bounded by systems of inequalities.
- Prove and disprove triangles and quadrilaterals given set of points.
- Prove and disprove perpendicularity given a set of points.
- Prove and disprove parallel lines.
- Divide segments proportionally.
- Find a point on a directed line segment through partitioning.

Standards Addressed:

CCSS.MATH.CONTENT.HSG.CO.A.1
Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

CCSS.MATH.CONTENT.HSG.GPE.B.4
Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).

CCSS.MATH.CONTENT.HSG.GPE.B.5
Prove the slope criteria for parallel and perpendicular lines and use them to solve geometric problems (e.g., find the equation of a line parallel or perpendicular to a given line that passes through a given point).

CCSS.MATH.CONTENT.HSG.GPE.B.6
Find the point on a directed line segment between two given points that partitions the segment in a given ratio.
CCSS.MATH.CONTENT.HSG.GPE.B.7
Use coordinates to compute perimeters of polygons and areas of triangles and rectangles, e.g., using the distance formula.*

UNIT 2

Unit Title: Constructions and Transformations
Unit Description: The heart of the module is the study of transformations and the role transformations play in defining congruence. Students will learn the definitions of particular geometric figures and develop a deeper understanding of them through constructions and measurements. In this unit, students will develop definitions of rigid transformations and use them to define congruent figures.

LEARNING GOALS

Enduring Understanding(s):
- Constructions support concrete understandings of abstract ideas
- Transformations of rigid motions preserve distance and angle measure, which is the foundation for the understandings of congruency

Essential Question(s):
- Why will Geometric figures be transformed in a plane?
- How can we strategically apply a rigid motion to informally show that two triangles are congruent?
- How do we determine the corresponding parts of two different figures?

Content and Skills:
- Define basic rigid motions
- Use a compass to construct line segments, angles, and perpendicular and parallel lines
- Use a compass to bisect line segments and angles
- Use a compass to construct an equilateral triangle
- Use a compass to transform figures by reflection, translation, and rotations
- Use a compass to construct a line of reflection and center of rotation
- Use a compass and/or protractor to determine degrees of rotation*
- Explain a point, line segments and/or figure is equidistant from the line reflection.
- Predict a sequence of rigid motion that maps a preimage onto a congruent image or itself
- Conclude rigid motions preserve distance and angle measures.
- Map corresponding sides and angles of congruent figures using rigid motions.

Standards Addressed:
CCSS.MATH.CONTENT.HSG.CO.A.2
Represent transformations in the plane using, e.g., transparencies and geometry software; describe transformations as functions that take points in the plane as inputs and give other points as outputs. Compare transformations that preserve distance and angle to those that do not (e.g., translation versus horizontal stretch).
CCSS.MATH.CONTENT.HSG.CO.A.3
Given a rectangle, parallelogram, trapezoid, or regular polygon, describe the rotations and reflections that carry it onto itself.
CCSS.MATH.CONTENT.HSG.CO.A.4
Develop definitions of rotations, reflections, and translations in terms of angles, circles, perpendicular lines, parallel lines, and line segments.
CCSS.MATH.CONTENT.HSG.CO.A.5
Given a geometric figure and a rotation, reflection, or translation, draw the transformed figure using, e.g., graph paper, tracing paper, or geometry software. Specify a sequence of transformations that will carry a given figure onto another.

CCSS.MATH.CONTENT.HSG.CO.B.6
Use geometric descriptions of rigid motions to transform figures and to predict the effect of a given rigid motion on a given figure; given two figures, use the definition of congruence in terms of rigid motions to decide if they are congruent.
CCSS.MATH.CONTENT.HSG.CO.B.7
Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
CCSS.MATH.CONTENT.HSG.CO.D.12
Make formal geometric constructions with a variety of tools and methods (compass and straightedge, string, reflective devices, paper folding, dynamic geometric software, etc.). Copying a segment; copying an angle; bisecting a segment; bisecting an angle; constructing perpendicular lines, including the perpendicular bisector of a line segment; and constructing a line parallel to a given line through a point not on the line.
CCSS.MATH.CONTENT.HSG.CO.D.13
Construct an equilateral triangle, a square, and a regular hexagon inscribed in a circle

UNIT 3

Unit Title: Properties and Applications of Angles and Segments
Unit Description: In order to make sense of geometry, students must be proficient with certain skills that involve equations. Specifically, they must be able to evaluate expressions, write and solve linear and quadratic equations, solve a system of equations. Students must be proficient in understanding relationships between angles and parallel lines cut by a transversal. Students will use the knowledge gained from the constructions unit to prove definitions in order model and solve real world applications.

LEARNING GOALS

Enduring Understanding(s):
- Geometric relationships and constructions are used to model problems in many real world fields such as: Building and Construction, Engineering Textile Design, Hobbies such as model building, Art
- Angle and segment relationships can be described and explored as functional relationships.
- Mathematical statements can be justified through deductive and inductive reasoning and proof.

Essential Question(s):
- How do geometric relationships and constructions help us to solve problems and make sense of our world?
- How is proof an important tool in everyday life?
- How can you describe the attributes of 2 or more angles based on their relationship?

Content and Skills:
- Define angle relationships when parallel lines are cut by a transversal.
- Determine if angles are congruent or supplementary based on relationship.
- Solve problems involving vertical angles, supplementary angles, linear pairs and complementary angles.
- Prove and disprove parallel lines using angle relationships.
- Solve problems using systems of equations and factoring.*
- Solve problems using segment addition property and angle addition property.
- Prove linear and angular relationships using known geometric reasoning.
- Construct auxiliary line to assist in solving problems.

Standards Addressed:
CCSS.MATH.CONTENT.HSG.CO.A.1
Know precise definitions of angle, circle, perpendicular line, parallel line, and line segment, based on the undefined notions of point, line, distance along a line, and distance around a circular arc.

CCSS.MATH.CONTENT.HSG.CO.C.9
Prove theorems about lines and angles. Theorems include: vertical angles are congruent; when a transversal crosses parallel lines, alternate interior angles are congruent and corresponding angles are congruent; points on a perpendicular bisector of a line segment are exactly those equidistant from the segment's endpoints.

UNIT 4

Unit Title: Congruence, Proofs and Triangles
Unit Description: Students will use deductive reasoning to construct arguments and write formal proofs for Geometric Theorems about lines, angles, triangles, and parallelograms.

LEARNING GOALS

Enduring Understanding(s):
- Understanding mathematical proof can help us evaluate arguments and arrive at meaningful conclusions in a variety of real world situations including political rhetoric, current events, advertising and finance.
- Mathematical statements can be justified through deductive and inductive reasoning and proof.

Essential Question(s):
- How are properties, postulates and theorems used in proofs in geometry?
- How do you use deductive reasoning to draw conclusions?

Content and Skills:
- Classify triangles based on side length and angle measure.
- Prove triangles congruent using SSS, SAS, AAS, ASA and HL
- Prove segments and angles congruent using CPCTC
- Justify congruent triangle (informally show congruent triangles using reasoning)
- Determine additional information necessary for proofs.
- Define a centroid, median, altitude, height, incenter*, circumcenter* within triangles.
- Solve problems involving a centroid, median, altitude, height, incenter*, circumcenter* within triangles.

Standards Addressed:
CCSS.MATH.CONTENT.HSG.CO.B.7
Use the definition of congruence in terms of rigid motions to show that two triangles are congruent if and only if corresponding pairs of sides and corresponding pairs of angles are congruent.
CCSS.MATH.CONTENT.HSG.CO.B.8
Explain how the criteria for triangle congruence (ASA, SAS, and SSS) follow from the definition of congruence in terms of rigid motions.
CCSS.MATH.CONTENT.HSG.CO.C.10
Prove theorems about triangles. Theorems include: measures of interior angles of a triangle sum to 180°; base angles of isosceles triangles are congruent; the segment joining midpoints of two sides of a triangle is parallel to the third side and half the length; the medians of a triangle meet at a point.

UNIT 5

Unit Title: Similarity

Unit Description: To be able to define similarity, there must be a definition of similarity transformations and consequently a definition for dilations. Students are introduced to the progression of terms beginning with scale drawings, which they first study in Grade 7 (Module 1, Topic D), but in a more observational capacity than in Grade 10: Students determine the scale factor between a figure and a scale drawing or predict the lengths of a scale drawing, provided a figure and a scale factor.

LEARNING GOALS

Enduring Understanding(s):

- Changing the size of an object proportionally does not change the angle measures.
- Similarity relationships between objects are a form of proportional relationships seen and used in real
- world.(scale models)
- Accurate use of ratio proportions has important real world applications.
- Congruency described a special type of similarity relationship between objects and is a form of equivalence.

Essential Question(s):

- How are the ratios of similar figures used to find missing parts?
- How can I tell if two figures are similar?
- How do I recognize a proportional situation and how can I use proportions to solve problems?
- Why are scaled drawings important in the real world?

Content and Skills:

- Prove and disprove figures similar using SSS, SAS, AA
- Determine side lengths of triangles using scale factor
- Construct scale drawings using proportions
- Define dilations as a transformation that preserves only angle measures (not distance)
- Compare and contrast similarity and congruence
- Define midsegment and use it to solve problems
- Solve problems using sidesplitter (triangle proportionality) theorem
- Solve problems using angle bisector theorem
- Solve proportions within right triangles (geometric mean)
- Solve realworld problems using proportions
- Compare proportional relationships of area and perimeter of similar figures

Standards Addressed:

CCSS.MATH.CONTENT.HSG.SRT.A.1

Verify experimentally the properties of dilations given by a center and a scale factor:

CCSS.MATH.CONTENT.HSG.SRT.A.2

Given two figures, use the definition of similarity in terms of similarity transformations to decide if they are similar; explain using similarity transformations the meaning of similarity for triangles as the equality of all corresponding pairs of angles and the proportionality of all corresponding pairs of sides.

CCSS.MATH.CONTENT.HSG.SRT.A.3

Use the properties of similarity transformations to establish the AA criterion for two triangles to be similar. Prove theorems involving similarity

CCSS.MATH.CONTENT.HSG.SRT.B.4

Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

CCSS.MATH.CONTENT.HSG.SRT.B.5

Use congruence and similarity criteria for triangles to solve problems and to prove relationships in geometric figures.

CCSS.MATH.CONTENT.HSG.MG.A.1

Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human

torso as a cylinder).*

UNIT 6

Unit Title: Circles With and Without Coordinates

Unit Description: This unit brings together the ideas of similarity and congruence studied in previous units and the work of geometric construction studied throughout the entire year thus far. It also includes the specific properties of triangles, special quadrilaterals, parallel lines and transversals, and rigid motions established and built upon throughout this course of study. This module's focus is on the possible geometric relationships between a pair of intersecting lines and a circle drawn on the page. If the lines are perpendicular and one passes through the center of the circle, then the relationship encompasses the perpendicular bisectors of chords in a circle and the association between a tangent line and a radius drawn to the point of contact. If the lines meet at a point on the circle, then the relationship involves inscribed angles. If the lines meet at the center of the circle, then the relationship involves central angles. If the lines meet at a different point inside the circle or at a point outside the circle, then the relationship includes the secant angle theorems and tangent angle theorems.

LEARNING GOALS

Enduring Understanding(s):

- Analyzing geometric relationships develops critical thinking skills.
- Real world problems can be modeled using circles and their properties.
- Predictions can be made in the real world using inductive reasoning based on circles.

Essential Question(s):

- What makes circles similar?
- How does constructing inscribed and circumscribed circles around a triangle prove properties of angles for a quadrilateral inscribed in a circle?
- What is the equation of a circle?
- What are the differences between angles, arcs and segments within a circle?
- What properties do all circles have?
- How are circles used in the real world?

Content and Skills:

- Identify and define circle vocabulary
- Explain why all circles are similar
- Solve problems using properties of arcs and angles
- Solve problems using theorems of segments insides and outside circles
- Solve problems using properties of tangent lines
- Write the equation of a circle
- Use equations of circles to prove points that are contained on circles
- Use the distance formula to construct circles
- Solve for measures of inscribed angles and inscribed polygons
- Model real world problems involving circles
- Determine arc length and area of a sector in degrees

Standards Addressed:

CCSS.MATH.CONTENT.HSG.C.A.1
Prove that all circles are similar.
CCSS.MATH.CONTENT.HSG.C.A.2
Identify and describe relationships among inscribed angles, radii, and chords. Include the relationship between central, inscribed, and circumscribed angles; inscribed angles on a diameter are right angles; the radius of a circle is

perpendicular to the tangent where the radius intersects the circle.
CCSS.MATH.CONTENT.HSG.C.A.3
Construct the inscribed and circumscribed circles of a triangle, and prove properties of angles for a quadrilateral inscribed in a circle.
CCSS.MATH.CONTENT.HSG.GPE.A.1
Derive the equation of a circle of given center and radius using the Pythagorean Theorem; complete the square to find the center and radius of a circle given by an equation.
CCSS.MATH.CONTENT.HSG.GPE.B.4
Use coordinates to prove simple geometric theorems algebraically. For example, prove or disprove that a figure defined by four given points in the coordinate plane is a rectangle; prove or disprove that the point $(1, \sqrt{3})$ lies on the circle centered at the origin and containing the point (0, 2).
CCSS.MATH.CONTENT.HSG.MG.A.1
Use geometric shapes, their measures, and their properties to describe objects (e.g., modeling a tree trunk or a human torso as a cylinder).*

UNIT 7

Unit Title: Trigonometry
Unit Description: Students are laying the foundation to studying trigonometry by focusing on similarity between right triangles in particular (the importance of the values of corresponding length ratios between similar triangles is particularly apparent in Lessons 16, 21, and 25). Students discover that a right triangle can be divided into two similar subtriangles (MP.2) to prove the Pythagorean theorem (GSRT.B.4). Two lessons are spent studying the algebra of radicals that is useful for solving for sides of a right triangle and computing trigonometric ratios.

LEARNING GOALS

Enduring Understanding(s):
- Trig ratios are a constant of proportionality
- Trigonometric relationships can be described and explored as functional relationships.

Essential Question(s):
- How can ratios within figures be used to find lengths of sides in another triangle when the two are known to be similar?
- How are expressions and equations that contain radicals simplified or solved?
- How do you find a side length or angle measure of a right triangle?
- How do trigonometric ratios relate to similar right triangles?

Content and Skills:
- Define trigonometric relationships within right triangles (Sine, cosine, tangent)
- Prove sine of an angle is equal to the cosine of its complement
- Derive trig ratios from similar right triangles
- Find missing side lengths of right triangles
- Find missing angles of right triangles
- Determine when to use angle of elevation versus angle of depression in real life situation
- Demonstrate the relationship between angle of elevation and depression to parallel lines cut by a transversal
- Determine when to use SohCahToa and when to use Law of sines and cosines**

Standards Addressed:
CCSS.MATH.CONTENT.HSG.SRT.B.4
Prove theorems about triangles. Theorems include: a line parallel to one side of a triangle divides the other two proportionally, and conversely; the Pythagorean Theorem proved using triangle similarity.

CCSS.MATH.CONTENT.HSG.SRT.C.6
Understand that by similarity, side ratios in right triangles are properties of the angles in the triangle, leading to definitions of trigonometric ratios for acute angles.
CCSS.MATH.CONTENT.HSG.SRT.C.7
Explain and use the relationship between the sine and cosine of complementary angles.
CCSS.MATH.CONTENT.HSG.SRT.C.8
Use trigonometric ratios and the Pythagorean Theorem to solve right triangles in applied problems.*

UNIT 8

Unit Title: The Unit Circle
Unit Description: Students are introduced to the unit circle and the trigonometric functions associated with it.

LEARNING GOALS

Enduring Understanding(s):
- Extend the knowledge of circles and trigonometry to form new connections.
- Multiple representations of measurement can be applied to given situations.

Essential Question(s):
- How do radians and degrees relate?
- How do you find area of a sector?
- What are the properties of the unit circle?
- How does trig relate to the unit circle?

Content and Skills:
- Convert from degrees to radians and vice versa
- Define tangent as a ratio of sine and cosine
- Determine where the values of tangent are undefined
- Find the arc length and area of a sector in radians
- Interpret the unit circle using trigonometric understandings
- Solve problems using the unit circle
- Demonstrate the length of the arc on the unit circle is subtended by the angle
- Explain symmetry and periodicity of trigonometric functions*

Standards Addressed:
CCSS.MATH.CONTENT.HSG.C.B.5
Derive using similarity the fact that the length of the arc intercepted by an angle is proportional to the radius, and define the radian measure of the angle as the constant of proportionality; derive the formula for the area of a sector.
CCSS.MATH.CONTENT.HSF.TF.A.1
Understand radian measure of an angle as the length of the arc on the unit circle subtended by the angle.
CCSS.MATH.CONTENT.HSF.TF.A.2
Explain how the unit circle in the coordinate plane enables the extension of trigonometric functions to all real numbers, interpreted as radian measures of angles traversed counterclockwise around the unit circle.
CCSS.MATH.CONTENT.HSF.TF.A.3
(+) Use special triangles to determine geometrically the values of sine, cosine, tangent for $\pi/3$, $\pi/4$ and $\pi/6$, and use the unit circle to express the values of sine, cosine, and tangent for x, $\pi + x$, and 2π x
in terms of their values for x, where x
is any real number.
CCSS.MATH.CONTENT.HSF.TF.A.4
(+) Use the unit circle to explain symmetry (odd and even) and periodicity of trigonometric functions.

UNIT 9

Unit Title: Three Dimensions

Unit Description: Students' knowledge of two-dimensional and three-dimensional objects is extended to include explanations of circumference and perimeter of the base of three dimensional objects in order to develop surface area and volume formulas. Additionally, students apply their knowledge of two-dimensional shapes to consider the shapes of cross-sections and the result of rotating a two-dimensional object about a line.

LEARNING GOALS

Enduring Understanding(s):

- Dimensional relationships can be used to determine space within or around a solid.
- Volume and surface area must be differentiated between when applied to a real world situation.

Essential Question(s):

- How can you determine the intersection of a solid and a plane?
- How do you find the surface area and volume of a solid?
- How do the surface areas and volumes of similar solids compare?

Content and Skills:

- Define cross section as an intersection of a plane and a solid that is parallel to the base(s).
- Find area of cross sections of solids
- Find the surface area of prisms, pyramids, cylinders, cones and spheres
- Find volume of prisms, pyramids, cylinders, cones and spheres
- Use concepts of density based on area and volume in modeling situations
- Solve real life problems using volume and surface area

Standards Addressed:

CCSS.MATH.CONTENT.HSG.GMD.A.1

Give an informal argument for the formulas for the circumference of a circle, area of a circle, volume of a cylinder, pyramid, and cone. Use dissection arguments, Cavalieri's principle, and informal limit arguments.

CCSS.MATH.CONTENT.HSG.GMD.A.3

Use volume formulas for cylinders, pyramids, cones, and spheres to solve problems.* Visualize relationships between two-dimensional and three-dimensional objects

CCSS.MATH.CONTENT.HSG.GMD.B.4

Identify the shapes of two-dimensional Cross-sections of three-dimensional objects, and identify three-dimensional objects generated by rotations of two-dimensional objects.

CCSS.MATH.CONTENT.HSG.MG.A.2

Apply concepts of density based on area and volume in modeling situations (e.g., persons per square mile, BTUs per cubic foot).*