Milford Public Schools Curriculum

Department: Mathematics
Course Name: Algebra II

UNIT 1

Unit Title: Functions
Unit Description: Students learn the formal definition of a function and how to recognize, evaluate, and interpret functions in abstract and contextual situations (**FIF.A.1**, **FIF.A.2**). Students examine the graphs of a variety of functions and learn to interpret those grhs using precise terminology to describe such key features as domain and range, intercepts, intervals where the function is increasing or decreasing, and intervals where the function is positive or negative. (FIF.A.1, FIF.B.4, FIF.B.5, FIF.C.7a).

LEARNING GOALS

Enduring Understanding(s):

- Piecewise functions are multiple functions being graphed over specific portions of the coordinate plane.
- The properties of functions and function operations are used to model and analyze real-world applications and quantitative relationships.

Essential Question(s):

- What is a function and how is the notation useful in math?
- What defines a function?

Content and Skills:

- Write and understand expressions using function notation f(x)
- Compose functions and describe the domain and range
- Apply their understanding of transformations of functions and their graphs to piecewise functions.
- Graph piecewise functions on a coordinate plan and describe the domain and range
- Discover the inverse of a function and describe the properties of the inverse and original function.

Standards Addressed:
CCSS.MATH.CONTENT.HSF.IF.A.1
Understand that a function from one set (called the domain) to another set (called the range) assigns to each element of the domain exactly one element of the range. If f is a function and x is an element of its domain, then f(x) denotes the output of f corresponding to the input x. The graph of f is the graph of the equation y = f(x).
CCSS.MATH.CONTENT.HSF.IF.A.2
Use function notation, evaluate functions for inputs in their domains, and interpret statements that use function notation in terms of a context.
CCSS.MATH.CONTENT.HSF.IF.C.7.B
Graph square root, cube root, and piece-wise defined functions, including step functions and absolute value functions.
CCSS.MATH.CONTENT.HSF.BF.B.4
Find inverse functions.
CCSS.MATH.CONTENT.HSF.BF.B.4.A
Solve an equation of the form f(x) = c for a simple function f that has an inverse and write an expression for the inverse. For example, f(x) =2 x3 or f(x) = (x+1)/(x1)
for x ≠ 1.
Level 1
CCSS.MATH.CONTENT.HSF.BF.A.1.C
(+) Compose functions. For example, if T(y) is the temperature in the atmosphere as a function of height, and h(t) is the height of a weather balloon as a function of time, then T(h(t)) is the temperature at the location of the weather balloon as a function of time.
CCSS.MATH.CONTENT.HSF.BF.B.4.B

(+) Verify by composition that one function is the inverse of another.
CCSS.MATH.CONTENT.HSF.BF.B.4.C
(+) Read values of an inverse function from a graph or a table, given that the function has an inverse.

UNIT 2

Unit Title: Quadratics
Unit Description: This unit focuses on deriving the analytic equation for a parabola given the focus and directrix (G.GPE.A.2) and showing that it is a quadratic equation. In doing so, students are able to tie together many powerful ideas from geometry and algebra, including transformations, coordinate geometry, polynomial equations, the Pythagorean Theorem, and functions. Students extend their facility with finding zeros of polynomials to include complex zeros. Students begin by solving systems of linear and nonlinear equations to which no real solutions exist, and then relate this to the possibility of quadratic equations with no real solutions. Students discover that complex numbers have real uses; in fact, they can be used in finding real solutions of polynomial equations. Students develop facility with p

LEARNING GOALS

Enduring Understanding(s):
- There are a variety of strategies to solve quadratic equations.
- There a variety of forms for quadratic equations and each form serves a different purpose.

Essential Question(s):
- How do you decide which strategy to use when solving a quadratic function?
- What are the advantages of a quadratic function in vertex form? In standard form?
- How are the real solutions of a quadratic equation related to the graph of the related quadratic function?

Content and Skills:
- Utilize different methods to factor quadratics of different forms.
- Apply their knowledge of the zero product property to solve quadratic equations.
- Simplifying Square Root expressions to obtain exact solutions of quadratics when using the quadratic formula.
- Identify when using the quadratic formula is an appropriate method for solving.
- Solve quadratic equations by completing the square
- Determine whether quadratics are in standard or vertex form and graph them appropriately.
- Apply their knowledge of all aspects of quadratics to solve modeling and real world application problems.

Standards Addressed:
CCSS.MATH.CONTENT.HSA.SSE.B.3
Choose and produce an equivalent form of an expression to reveal and explain properties of the quantity represented by the expression.*
CCSS.MATH.CONTENT.HSA.SSE.B.3.A
Factor a quadratic expression to reveal the zeros of the function it defines.
CCSS.MATH.CONTENT.HSA.SSE.B.3.B
Complete the square in a quadratic expression to reveal the maximum or minimum value of the function it defines.
CCSS.MATH.CONTENT.HSA.SSE.A.2
Use the structure of an expression to identify ways to rewrite it.
CCSS.MATH.CONTENT.HSA.REI.B.4
Solve quadratic equations in one variable.
CCSS.MATH.CONTENT.HSA.REI.B.4.A
Use the method of completing the square to transform any quadratic equation in x into an equation of the form (x p)
2 = q that has the same solutions. Derive the quadratic formula from this form.
CCSS.MATH.CONTENT.HSA.REI.B.4.B

Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.
CCSS.MATH.CONTENT.HSA.REI.D.10
Understand that the graph of an equation in two variables is the set of all its solutions plotted in the coordinate plane, often forming a curve (which could be a line).
CCSS.MATH.CONTENT.HSG.GPE.A.2
Derive the equation of a parabola given a focus and directrix.
CCSS.MATH.CONTENT.HSF.IF.C.7
Graph functions expressed symbolically and show key features of the graph, by hand in simple cases and using technology for more complicated cases.*
CCSS.MATH.CONTENT.HSF.IF.C.7.A
Graph linear and quadratic functions and show intercepts, maxima, and minima.
CCSS.MATH.CONTENT.HSF.IF.C.8
Write a function defined by an expression in different but equivalent forms to reveal and explain different properties of the function.
CCSS.MATH.CONTENT.HSF.IF.C.8.A
Use the process of factoring and completing the square in a quadratic function to show zeros, extreme values, and symmetry of the graph, and interpret these in terms of a context.

UNIT 3

Unit Title: Polynomials
Unit Description: Students connect polynomial arithmetic to computations with whole numbers and integers. Students learn that the arithmetic of rational expressions is governed by the same rules as the arithmetic of rational numbers. This unit helps students see connections between solutions to polynomial equations, zeros of polynomials, and graphs of polynomial functions. Polynomial equations are solved over the set of complex numbers, leading to a beginning understanding of the fundamental theorem of algebra. Application and modeling problems connect multiple representations and include both real world and purely mathematical situations.

LEARNING GOALS

Enduring Understanding(s):

- The zeros and multiplicities of a polynomial relate to the equation of the polynomial and end behavior
- Polynomials can be used to model real world situations

Essential Question(s):

- What information can be determined from a polynomial function?
- What is the relationship between factors, roots, zeros, and xintercepts and how do they relate to the associated polynomial equation?
- What type of polynomial function is appropriate for modeling real world situations?

Content and Skills:
Students will be able to:

- apply their knowledge of multiplying binomials to multiply polynomials.
- divide polynomials with or without a remainder (long, tabular, and synthetic) to find the zeroes of the polynomial
- add and subtract polynomials using prior knowledge of combining like terms.
- factor polynomials by grouping by extending their knowledge of splitting the middle term.
- expand their knowledge of factoring Special Cases by applying this skill to polynomial expressions.
- graph factored polynomials and identify the multiplicities of each factor, degree, and end behavior of the function.
- properly utilize their graphing calculator to model polynomial functions.
- model with polynomials to apply them to real world situations.

Standards Addressed:
CCSS.MATH.CONTENT.HSA.APR.B.3
Identify zeros of polynomials when suitable factorizations are available, and use the zeros to construct a rough graph of the function defined by the polynomial
CCSS.MATH.CONTENT.HSA.APR.B.2
Know and apply the Remainder Theorem: For a polynomial p(x) and a number a, the remainder on division by x a is p(a), so p(a) = 0 if and only if (x a) is a factor of p(x).
CCSS.MATH.CONTENT.HSN.Q.A.2
Define appropriate quantities for the purpose of descriptive modeling.
CCSS.MATH.CONTENT.HSA.SSE.A.2
Use the structure of an expression to identify ways to rewrite it. For example, see x4 y4 as (x2)2 (y2)2, thus recognizing it as a difference of squares that can be factored as (x2 y2)(x2 + y2).
CCSS.MATH.CONTENT.HSA.APR.C.4
Prove polynomial identities and use them to describe numerical relationships. For example, the polynomial identity (x2 + y2)2 = (x2 y2) 2 + (2xy)2 can be used to generate Pythagorean triples.
CCSS.MATH.CONTENT.HSF.IF.C.7.C
Graph polynomial functions, identifying zeros when suitable factorizations are available, and showing end behavior.
Level 1 Extensions
CCSS.MATH.CONTENT.HSN.CN.C.8
(+) Extend polynomial identities to the complex numbers. For example, rewrite x2 + 4 as (x + 2i)(x 2i).
CCSS.MATH.CONTENT.HSN.CN.C.9
(+) Know the Fundamental Theorem of Algebra; show that it is true for quadratic polynomials.
CCSS.MATH.CONTENT.HSA.APR.D.7
(+) Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.

UNIT 4

Unit Title: Rational and Radical Expressions
Unit Description: Students solve rational, and radical equations, and apply these types of equations to real-world situations. They examine the conditions under which an extraneous solution is introduced. They rewrite rational expressions in different forms and work with radical expressions as part of this process. Students work with systems of equations that include quadratic and linear equations and apply their work to understanding the definition of a parabola. Students extend their facility with solving polynomial equations to working with complex zeros. Complex numbers are introduced via their relationship with geometric transformations. The unit concludes with students realizing that every polynomial function can be written as a product of linear factors, which is not possible without complex numbers.

LEARNING GOALS

Enduring Understanding(s):
- Rational expressions are similar to fractions
- A common denominator is needed to add and subtract rational expressions and that a common denominator is not needed for multiplication or division.
- There is a relationship between radicals and exponents.

Essential Question(s):
- How are operations with rational expressions like operations with fractions? How are they different?
- How do you solve equations with radical expressions?
- Why are extraneous solutions necessary to find?

Content and Skills:
Students will be able to:

- use imaginary and complex numbers to rationalize denominators.
- simplify *rational* expressions to
- extend their knowledge of fractions and cross cancelling to multiply and divide *rational* expressions
- translate their knowledge of common denominators to add and subtract *rational* expressions
- solve *rational* equations by clearing a common denominator.
- model with *rational* expressions
- develop previous knowledge of radical expressions to solve radical equations.
- explain what an extraneous solution is and why they are necessary. (within each topic)

Standards Addressed:
CCSS.MATH.CONTENT.HSN.CN.A.1
Know there is a complex number i such that i2 = 1, and every complex number has the form a + bi with a and b real.
CCSS.MATH.CONTENT.HSN.CN.A.2
Use the relation i2 = 1 and the commutative, associative, and distributive properties to add, subtract, and multiply complex numbers.
CCSS.MATH.CONTENT.HSN.CN.C.7
Solve quadratic equations with real coefficients that have complex solutions.
CCSS.MATH.CONTENT.HSA.APR.D.6
Rewrite simple rational expressions in different forms; write a(x)/b(x) in the form q(x) + r(x)/b(x), where a(x), b(x), q(x), and r(x) are polynomials with the degree of r(x) less than the degree of b(x), using inspection, long division, or, for the more complicated examples, a computer algebra system.
CCSS.MATH.CONTENT.HSA.APR.D.7
Understand that rational expressions form a system analogous to the rational numbers, closed under addition, subtraction, multiplication, and division by a nonzero rational expression; add, subtract, multiply, and divide rational expressions.
CCSS.MATH.CONTENT.HSA.REI.A.1
Explain each step in solving a simple equation as following from the equality of numbers asserted at the previous step, starting from the assumption that the original equation has a solution. Construct a viable argument to justify a solution method.
CCSS.MATH.CONTENT.HSA.REI.A.2
Solve simple rational and radical equations in one variable, and give examples showing how extraneous solutions may arise.
CCSS.MATH.CONTENT.HSA.REI.B.4.B
Solve quadratic equations by inspection (e.g., for x2 = 49), taking square roots, completing the square, the quadratic formula and factoring, as appropriate to the initial form of the equation. Recognize when the quadratic formula gives complex solutions and write them as a ± bi for real numbers a and b.
Level 1 Extensions
CCSS.MATH.CONTENT.HSN.CN.A.3
(+) Find the conjugate of a complex number; use conjugates to find moduli and quotients of complex numbers.

UNIT 5

Unit Title: Exponential and Logarithmic Equations
Unit Description: In this unit, students synthesize and generalize what they have learned about a variety of function families. They extend the domain of exponential functions to the entire real line (NRN.A.1) and then extend their work with these functions to include solving exponential equations with logarithms (FLE.
A.4). They explore (with appropriate tools) the effects of transformations on graphs of exponential and logarithmic functions. They notice that the transformations on a graph of a logarithmic function relate to the logarithmic properties (FBF.B.3). Students identify appropriate types of functions to model a situation. They adjust parameters to improve the model, and they compare models by analyzing appropriateness of fit and making judgments about the domain over which a model is a good fit. The description of modeling as, "the process of choosing and using mathematics and statistics to analyze empirical situations, to understand them better, and to make decisions," is at the heart of this module. In particular, through repeated opportunities in working through the modeling cycle (see page 61 of the CCLS), students acquire the insight that the same mathematical or statistical structure can sometimes model seemingly different situations. This unit builds on the work in Algebra 1, where students first modeled situations using exponential functions and considered which type of function would best model a given real world situation. The unit also introduces students to the extension standards relating to inverse functions and composition of functions to further enhance student understanding of logarithms. The unit culminates with a project spread out over several lessons where students consider applying their knowledge to financial literacy. They plan a budget, consider borrowing money to buy a car and a home, study paying off a credit card balance, and finally, decide how they could accumulate one million dollars.

LEARNING GOALS

Enduring Understanding(s):
- Exponential and logarithmic functions are related to one another
- There are a variety of methods to solve exponential and logarithmic functions

Essential Question(s):
- How do you model a quantity that changes regularly over time by the same percentage?
- How are exponential and logarithmic functions related?
- What type of real world situations model exponential growth or decay?
- How can you use different parts of the equation to transform your function?

Content and Skills:
Students will be able to:
- recall properties of exponents to solve exponential and logarithmic equations
- apply numbers in scientific notation to applications of logarithmic and exponential functions.
- rewrite numbers with rational exponents in radical form to be applied in properties of logarithms.
- graph exponential functions to visualize the growth or decay of an application.
- synthesize their knowledge of exponential growth and decay to compound interest, halflife
- and compounding continuously.
- utilize their knowledge of rational exponent to solve exponential equations.
- identify and apply properties of logarithms to condense and expand logarithmic expressions.
- extend their knowledge of logarithms to include natural logarithms
- use properties of exponents and logarithms to solve logarithmic equations.
- combine their knowledge of logarithms and exponentials to solve real world application problems.

Standards Addressed:
CCSS.MATH.CONTENT.HSN.RN.A.1
Explain how the definition of the meaning of rational exponents follows from extending the properties of integer exponents to those values, allowing for a notation for radicals in terms of rational exponents. For example, we define 51/3 to be the cube root of 5 because we want (51/3)3= 5(1/3)3 to hold, so (51/3)3 must equal 5.

CCSS.MATH.CONTENT.HSN.RN.A.2
Rewrite expressions involving radicals and rational exponents using the properties of exponents.
CCSS.MATH.CONTENT.HSN.Q.A.2
Define appropriate quantities for the purpose of descriptive modeling.
CCSS.MATH.CONTENT.HSA.CED.A.1
Create equations and inequalities in one variable and use them to solve problems. Include equations arising from linear and quadratic functions, and simple rational and exponential functions.
CCSS.MATH.CONTENT.HSA.REI.D.11
Explain why the xcoordinates of the points where the graphs of the equations y = f(x) and y = g(x) intersect are the solutions of the equation f(x) = g(x); find the solutions approximately, e.g., using technology to graph the functions, make tables of values, or find successive approximations. Include cases where f(x) and/or g(x) are linear, polynomial, rational, absolute value, exponential, and logarithmic functions.*
CCSS.MATH.CONTENT.HSF.IF.B.4
For a function that models a relationship between two quantities, interpret key features of graphs and tables in terms of the quantities, and sketch graphs showing key features given a verbal description of the relationship.
CCSS.MATH.CONTENT.HSF.IF.C.7.E
Graph exponential and logarithmic functions, showing intercepts and end behavior, and trigonometric functions, showing period, midline, and amplitude.
CCSS.MATH.CONTENT.HSF.IF.C.8.B
Use the properties of exponents to interpret expressions for exponential functions. For example, identify percent rate of change in functions such as $y = (1.02)^t$, $y = (0.97)^t$, $y = (1.01)12^t$, $y = (1.2)^t/10$,
and classify them as representing exponential growth or decay.
CCSS.MATH.CONTENT.HSF.IF.C.9
Compare properties of two functions each represented in a different way (algebraically, graphically, numerically in tables, or by verbal descriptions).For example, given a graph of one quadratic function and an algebraic expression for another, say which has the larger maximum.
CCSS.MATH.CONTENT.HSF.BF.A.1
Write a function that describes a relationship between two quantities.*
CCSS.MATH.CONTENT.HSF.LE.A.2
Construct linear and exponential functions, including arithmetic and geometric sequences, given a graph, a description of a relationship, or two input-output pairs (include reading these from a table).
CCSS.MATH.CONTENT.HSF.LE.A.4
For exponential models, express as a logarithm the solution to abct = d where a, c, and d are numbers and the base b is 2, 10, or e; evaluate the logarithm using technology.
CCSS.MATH.CONTENT.HSF.LE.B.5
Interpret the parameters in a linear or exponential function in terms of a context.
Level 1 only
CCSS.MATH.CONTENT.HSF.BF.B.5
(+) Understand the inverse relationship between exponents and logarithms and use this relationship to solve problems involving logarithms and exponents.

UNIT 6

Unit Title: Inferential Statistics

Unit Description: The concepts of probability and statistics covered in this course build on students' previous work in Grades 7 and 9. Fundamental ideas from Grade 7 are revisited and extended to allow students to build a more formal understanding of probability. More complex events are considered (unions, intersections, complements) (**SCP.A.1**). Students calculate probabilities based on two-way data tables and interpret them in context (**SCP.A.4**). They also see how to create "hypothetical 1000" two-way tables as a way of calculating probabilities. Students are introduced to conditional probability (**SCP.A.3**, **SCP.A.5**), and the important concept of independence is developed (**SCP.A.2**, **SCP.A.5**). Students learn about the idea of using a smooth curve to model a data distribution, describes properties of the normal distribution, and asks students to distinguish between data distributions for which it would be reasonable to use a normal distribution as a model and those for which a normal distribution would not be a reasonable model. Students use tables and technology to find areas under a normal curve and interpret these areas in the context of modeling a data distribution (**SID.A.4**). This unit also develops students' understanding of statistical inference and introduce different types of statistical studies(observational studies, surveys, and experiments) (**SIC.B.3**). Students explore using data from a random sample to estimate a population mean or a population proportion. Building on what they learned about sampling variability in Grade 7, students use simulation to create an understanding of margin of error. Students calculate the margin of error and interpret it in context (**SIC.B.4**). Students also evaluate reports from the media using sample data to estimate a population mean or proportion (**SIC.B.6**). This unit also focuses on drawing conclusions based on data from a statistical experiment. Given data from a statistical experiment, students use simulation to create a randomization distribution and use it to determine if there is a significant difference between two treatments (**SIC.B.5**). Students also critique and evaluate published reports based on statistical experiments that compare two treatments (**SIC. B.6**).

LEARNING GOALS

Enduring Understanding(s):

- Different types of visual representations can be used to model a data set.
- Different types of sampling methods are more appropriate for different populations.
- Probability is dependent on all aspects of a scenario and you must include all potential outcomes.

Essential Question(s):

- What method can be used to measure a spread of data?
- What type of sample methods can be used to make conclusions about a population?
- How do you recognize bias in sampling methods used to make inferences about populations?
- How can the graph of a normal distribution of data help you understand the data?
- How do the potential outcomes of an event reflect the probability of a specific event occurring?

Content and Skills:

Students will be able to:

- Create a graph of a sample of data in the form of a histogram, dot plot, and boxplot.
- Model different distributions of data to analyze a data set.
- Analyze a data sample and draw conclusions from it
- Analyze an experimental data set and draw conclusions.
- Determine the probability of a specific event occurring for different scenarios.

Standards Addressed:

CCSS.MATH.CONTENT.HSS.ID.A.1
Represent data with plots on the real number line (dot plots, histograms, and box plots).
CCSS.MATH.CONTENT.HSS.ID.A.2
Use statistics appropriate to the shape of the data distribution to compare center (median, mean) and spread (interquartile range, standard deviation) of two or more different data sets.

CCSS.MATH.CONTENT.HSS.ID.A.3
Interpret differences in shape, center, and spread in the context of the data sets, accounting for possible effects of extreme data points (outliers).
CCSS.MATH.CONTENT.HSS.ID.A.4
Use the mean and standard deviation of a data set to fit it to a normal distribution and to estimate population percentages. Recognize that there are data sets for which such a procedure is not appropriate. Use calculators, spreadsheets, and tables to estimate areas under the normal curve.
CCSS.MATH.CONTENT.HSS.ID.B.5
Summarize categorical data for two categories in two-way frequency tables. Interpret relative frequencies in the context of the data (including joint, marginal, and conditional relative frequencies). Recognize possible associations and trends in the data.
CCSS.MATH.CONTENT.HSS.ID.B.6
Represent data on two quantitative variables on a scatter plot, and describe how the variables are related.
CCSS.MATH.CONTENT.HSS.CP.A.1
Describe events as subsets of a sample space (the set of outcomes) using characteristics (or categories) of the outcomes, or as unions, intersections, or complements of other events ("or," "and," "not").
CCSS.MATH.CONTENT.HSS.CP.A.2
Understand that two events A and B are independent if the probability of A and B occurring together is the product of their probabilities, and use this characterization to determine if they are independent.
CCSS.MATH.CONTENT.HSS.CP.A.3
Understand the conditional probability of A given B as P(A and B)/P(B), and interpret independence of A and B as saying that the conditional probability of A given B is the same as the probability of A, and the conditional probability of B given A is the same as the probability of B.
CCSS.MATH.CONTENT.HSS.CP.A.4
Construct and interpret two-way frequency tables of data when two categories are associated with each object being classified. Use the two-way table as a sample space to decide if events are independent and to approximate conditional probabilities.
CCSS.MATH.CONTENT.HSS.CP.A.5
Recognize and explain the concepts of conditional probability and independence in everyday language and everyday situations. For example, compare the chance of having lung cancer if you are a smoker with the chance of being a smoker if you have lung cancer.
CCSS.MATH.CONTENT.HSS.CP.B.6
Find the conditional probability of A given B as the fraction of B's outcomes that also belong to A, and interpret the answer in terms of the model.
CCSS.MATH.CONTENT.HSS.CP.B.7
Apply the Addition Rule, P(A or B) = P(A) + P(B) P(A and B), and interpret the answer in terms of the model.